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ON THE TOWNSEND MECHANISM OF SPARK BREAKDOWN OF GASES

by

R.W. Crowe, J.K. Bragg and V.G. Thomas

Report No. RL-1011 January 1954

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Title Page

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On the Townsend Mechanism of Spark Breakdown of Gases 15-517

ABSTRACT

Two recent experiments have provided the Townsend theory of the electrical breakdown of gases with a renewed vitality. It now appears, as a result of these experiments, that the Townsend theory is applicable to uniform field breakdown over the entire range of Pδ (the product of the pressure and the gap length). In the present report, further

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~~CONCLUSIONS~~

support for this mechanism is found in a precise correlation of prebreakdown conduction data with breakdown data. In order to establish such a correlation, it is necessary to assume a functional form for the dependence of the first Townsend coefficient on the electric field. It is possible to show that the assumption has an approximate theoretical validity.

The Townsend criterion for breakdown is based upon a singularity that is found in the equation of current as a function of voltage, which conventionally is derived assuming that space charges are absent. A more precise investigation of this criterion, involving the inclusion of the effect of space charge, removes the singularity and gives a current that is finite at every point. Beyond a certain current, however, the slope of the curve of current versus

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"Abstract (continued)"

voltage becomes negative, indicating that the system is incapable of withstanding a voltage larger than that at which the negative characteristic develops. The breakdown voltage so established will not differ appreciably from that given by the Townsend equation, in most cases.

ON THE TOWNSEND MECHANISM OF SPARK BREAKDOWN OF GASES

R.W. Crowe, J.K. Bragg and V.G. Thomas

I. INTRODUCTION

At one time, the so-called Townsend-type mechanisms of electrical breakdown of gases were supposed to be universally applicable to sparking in uniform fields. In the nineteen thirties, however, a number of experiments were performed that were widely interpreted as demonstrating that such mechanisms were applicable only when the product $P\delta$ was less than about 200 mm Hg x cm, P being the gas pressure and δ the electrode separation.* For example, the observation of "streamers" in positive point-to-plane corona(1) was taken to mean that such a phenomenon was also to be anticipated in uniform fields.(2) Furthermore, inaccurate experimental results indicated that the currents predicted by the Townsend equations were not in accord with reality when $P\delta$ was greater than about 200 in air; there were also certain erroneous interpretations of formative time lags. These circumstances led a number of workers to postulate so-called "streamer" mechanisms for uniform field breakdown, culminating in the quantitative theory of Meek.(3)

In 1951 and 1952, two separate and decisive experiments showed that the whole ten-year excursion into streamer theory was unnecessary. One of these(4) showed that the formative time lags observed when the product $P\delta$ is large are in agreement with the Townsend theory (this will be discussed in a separate report). The other(5) involved a study of the steady-state current versus δ at constant field. The Townsend theory predicts a certain curvature in the plot of the logarithm of the steady-state current against δ as δ increases, this curvature being caused by the intercession of a secondary mechanism. Measurements performed in 1933(6) at large values of $P\delta$ had failed to disclose such a curvature, and were therefore interpreted as negating the Townsend hypothesis. However, the experiments of Llewellyn Jones _et al_.(5) demonstrated the existence of the curvature demanded by the Townsend theory.

The present work supplies two necessary elements in the complete framework of the Townsend theory. First, it is demonstrated that sparking phenomena measured well above $P\delta$ = 200 can be correlated precisely with prebreakdown current measurements by the use of a Townsend mechanism. Second, the build-up of positive space-charge that has always been assumed to occur either at or beyond breakdown is shown to be described by a modification of the Townsend equation, which recognizes the presence of space charge and shows that the build-up actually occurs as the voltage approaches the threshold voltage predicted by the Townsend equation. Thus, the Townsend equation must be regarded as a very good approximation based upon correct physical hypotheses. Before we enter upon a detailed report of our investigations, however, the Townsend theory will be summarized.

---

*The circumstance that P and δ enter expressions for the sparking voltage only as the product $P\delta$ is the substance of Paschen's similarity law.

The two general processes that characterize a Townsend mechanism, as the term is used in this report, are the production of electrons by collision ionization throughout the gas and the production of secondary electrons at the cathode by ion or photon bombardment. The first is described by a coefficient α, that gives the number of ionizing collisions per centimeter made by an electron in its progress through the gas. The second is described by a generalized secondary ionization coefficient ω/α,(7) which is equal to a linear combination of γ (the fraction of ions falling on the cathode that produces secondary electrons) and β/α, where β is the number of photoelectrons produced at the cathode by photons created by an electron per centimeter of travel through the gas. With these definitions, the steady-state current flowing in a uniform-field gap can be shown to be:(8)

$$i = i_0 \exp \alpha\delta / [1-(\omega/\alpha)(\exp \alpha\delta - 1)]. \qquad (1)$$

This is the Townsend equation; in its use, ω/α is usually assumed to be independent of the electric field E, although α is an increasing function of E. The current thus increases with E toward an unbounded value reached when

$$(\omega/\alpha)(\exp \alpha\delta - 1) = 1, \qquad (2)$$

this condition being taken to define implicitly the breakdown (threshold) electric field. Because of the dependence of α on the electric field, these equations apply only when space charge is absent. Other secondary mechanisms have been considered in the past, but will not be treated here; in any event, their effects can be incorporated linearly in ω/α.

II. THE FUNCTIONAL FORM OF THE TOWNSEND α

It has been indicated that the dependence of the current on electric field is contained in the function $\alpha(E)$. In order to test the Townsend mechanism thoroughly, its predictions as to threshold voltage over a wide range of $P\delta$ must be compared with experiment, and this necessitates a knowledge of the functional form of $\alpha(E)$.

The first attempt to derive such a result was that of Townsend,(9) based on a very simple model; the calculation will be reproduced here because of its simplicity. If the fraction of free-electron paths in the gas of length greater than x is $\exp(-x/\lambda)$, where λ is a mean free path independent of energy, then the fraction of paths over which the electron attains ionization energy is $\exp(-\epsilon_i/\lambda eE)$, where ϵ_i is the ionization energy. The number of paths per centimeter in this one-dimensional model is $1/\lambda$, so that one may expect α to have the form

$$\alpha = (1/\lambda) \exp(-\epsilon_i/\lambda Ee), \qquad \lambda eE \ll \epsilon_i ,$$

or

$$(\alpha/P) = A \exp(-BP/E), \qquad (3)$$

since λ is inversely proportional to P.

There have been a number of more elaborate studies of this problem, starting from the calculation of the energy distribution of the electrons in the presence of the electric field. Some are based on the assumption of a Maxwellian distribution with a modified electron temperature,(10) and lead to a result of the form of Eq. (3) for α, in which the constant B contains the ionization energy as a factor.

Smit(11) and Druyvesteyn(12) have studied the distribution in energy most thoroughly, solving the Boltzmann equation without any assumptions as to the form of the distribution function. The principal deficiency of these calculations lies in the assumptions made about the various scattering cross sections, whose energy dependence must be known if the calculation is to be really complete. Only in the case of the rare gases does the available experimental information come close to satisfying this requirement.

Nevertheless, it is possible to show from Smit's work that certain variations in the energy-dependence of the scattering cross sections do not affect the form of Eq. (3); that is, there is some theoretical justification for the use of Eq. (3) with experimentally determined constants. One can show, however, that the functional form of the inelastic scattering cross section does influence the way in which the ionization energy enters in the constant B. To provide a background for the derivation of α and for the study of the influence of the functional form of the cross section, we will summarize here Smit's work on the electron energy distribution, upon which our work will be based.

Smit considers the following processes:

1. elastic collisions of electrons with atoms;
2. inelastic collisions of electrons with atoms;
3. energy exchange with the electric field.

Because of each of these processes, there is a current of electrons in "energy space," and the net current is given by:

$$n'-n'' = (n'-n'')_{\text{elast}} + (n'-n'')_{\text{inelast}} + (n'-n'')_{\text{field}} \,, \tag{4}$$

where n' is the number of electrons per second passing the energy ϵ toward higher energies, and n'' the number passing toward lower energies. The entire current in energy space must vanish in the steady state.

The contribution of the elastic scattering is worked out on the basis of a constant, kinetic-theory mean free path λ, and an energy exchange dictated by the relationship of the masses of the colliding bodies. Now, even for the rare gases, this assumption is invalidated by the observed variation of the so-called Ramsauer cross section with energy, but we shall be able to show that this discrepancy does not materially influence the high-energy tail of the distribution if the electric field is sufficiently large. This result is essential for our work with molecules, since slow electrons will suffer inelastic collisions involving transfer of small amounts of energy to molecular vibrations. As Morse has shown,(13) the rotation-vibration excitation of diatomic molecules contributes no more to the scattering than does the elastic scattering.

Smit's result for this part of the calculation is:

$$(n'-n'')_{\text{elast}} = \frac{4N}{3}\sqrt{\frac{2}{m}}\left(\frac{m}{M}\right)\bar{\epsilon}_a\,\epsilon^{3/2}\left[\left(\frac{1}{2\epsilon} - \frac{3}{2\bar{\epsilon}_a}\right)W - \frac{dW}{d\epsilon}\right] . \tag{5}$$

Here W is the probability distribution in energy; m and M are the masses of electron and atom, respectively; ϵ is the electron energy; $\bar{\epsilon}_a$ is the mean molecular kinetic energy; and N is the total number of electrons per unit volume.

The inelastic scattering considered by Smit is the excitation of atomic electrons by collision. It is assumed that practically all electrons that gain energy past ϵ_e (the threshold energy for excitation of atomic electrons) make an excitation collision before gaining ϵ^* electron volts of energy, ϵ^* being of the order of 1. Since the electron will suffer many

elastic collisions in this interval, the assumption is reasonable. Almost all the electrons that make excitation collisions will thus suffer a discontinuous loss of energy to an energy below ϵ^* electron volts. Thus, we may say:

$$(n'-n'')_{\text{inelast}} \cong -(n'')_{\text{inelast}} = -\sqrt{2/m}\;NN_a\int_{\epsilon}^{\infty}\sigma(\xi)\,\xi^{1/2}\,W(\xi)\,d\xi \,, \tag{6}$$

provided that $\epsilon > \epsilon^*$. In this equation, σ is the cross section for excitation (in general, a function of energy which is zero for $\epsilon < \epsilon_e$), and N_a is the number of molecules per unit volume.

Finally, the field terms are introduced with the use of the same assumptions regarding mean free path that were used in connection with the calculation of the elastic collision terms. The result is:

$$(n'-n'')_{\text{field}} = \sqrt{2/m}\;N\left[(1/6)\,e^2E^2\lambda\epsilon^{-1/2}\,W - (1/3)e^2E^2\lambda\epsilon^{1/2}(dW/d\epsilon)\right] . \tag{7}$$

The steady-state condition is then

$$F_0W + F_1\,(dW/d\epsilon) + G = 0, \tag{8}$$

where

$$F_0 = -F_1/2\epsilon + 2\,(m/M)\,(1/\lambda)\,\epsilon^{3/2} \quad ,$$

$$F_1 = (4/3)\,(m/M)\,(1/\lambda)\;\bar{\epsilon}_a\;\epsilon^{3/2} + (1/3)\,e^2E^2\lambda\epsilon^{1/2} \quad \text{, and}$$

$$G = N_a\int_{\epsilon}^{\infty}\xi^{1/2}\,\sigma\,W d\xi \quad .$$

The quantity G, by assumption, is constant for $\epsilon^* < \epsilon < \epsilon_e$.

The terms containing (m/M) are those arising from the elastic scattering. However, provided

$$(1/6)\,e^2E^2\lambda^2 >> 2(m/M)\epsilon^2 \quad , \tag{9}$$

that is, for sufficiently large E/P, they may be ignored.† Another way of phrasing our conclusions is that the vast majority of collisions (those not involving electronic excitation) are important only in determining the mobility of the electron.

---

†This was demonstrated by a calculation of W for an idealized system as a function of ϵ, including the influence of elastic scattering and an excitation cross section of the form $\sigma = \sigma_0(\epsilon - \epsilon_e)$. The results were almost identical to those calculated from Eqs. (11) and (14) below throughout the energy range of interest.

The differential Eq. (8) is thus vastly simplified; it becomes

$$dW/d\epsilon - (1/2\epsilon)\,W + G/a\epsilon^{1/2} = 0;\quad a = (1/3)\,e^2E^2\lambda. \tag{10}$$

In the range $\epsilon^* < \epsilon < \epsilon_e$ it is directly integrable, since G is constant. We have

$$W = \epsilon^{1/2}[C_1 - (G_o/a)\log\epsilon]\quad, \tag{11}$$

$$G_o = N_a\int_{\epsilon_e}^{\infty}\xi^{1/2}\sigma\,W d\xi\ .$$

In the range $\epsilon > \epsilon_e$ we differentiate Eq. (10), and neglect $1/4\epsilon^2$ in favor of $N_a\sigma/a$:

$$d^2W/d\epsilon^2 - (N_a\sigma/a)\,W = 0. \tag{12}$$

To integrate this equation, we must make some assumption as to the functional form of σ. Two choices will be illustrated. If $\sigma = \sigma_o$ = constant, then we have

$$W = C_2\exp(-\kappa\epsilon)\ ,\qquad \kappa = \sqrt{N_a\sigma_o/a}\quad; \tag{13}$$

while if

$$\sigma = \sigma_o(\epsilon - \epsilon_e),\ \text{we have}$$

$$W = C_3\sqrt{\eta}\,i^{4/3}H_{1/3}(2i\eta^{3/2}/3)\ , \tag{14}$$

where $H_{1/3}$ is a Hankel function and $\eta = (N_a\sigma_o/a)^{1/3}(\epsilon - \epsilon_e)$. Either Eq. (13) or (14) must be joined to Eq. (11) at ϵ_e, and the whole normalized, to give a solution for $W(\epsilon)$ over the entire range. It should be noted that ionization processes are assumed to occur at a rate slow enough not to perturb the distribution calculated with neglect of them.

To illustrate, let us consider Eq. (13), with which the above operations can be carried out explicitly. At $\epsilon = \epsilon_e$, we have

$$C_1 = C_2\exp(-\kappa\epsilon_e)\,\epsilon_e^{-1/2} + (G_o/a)\log\epsilon_e\ .$$

Then

$$W = \epsilon^{1/2}[C_2\exp(-\kappa\epsilon_e)\,\epsilon_e^{-1/2} - (G_o/a)\log(\epsilon/\epsilon_e)],\quad \epsilon<\epsilon_e\ . \tag{15}$$

Furthermore,

$$G_o = C_2N_a\int_{\epsilon_e}^{\infty}\xi^{1/2}\sigma_o\exp(-\kappa\xi)\,d\xi = (C_2N_a\epsilon_e^{1/2}/\kappa)\,\sigma_o\exp(-\kappa\epsilon_e)\ ,$$

or

$$W = C_2\exp(-\kappa\epsilon_e)\,(\epsilon/\epsilon_e)^{1/2}[1 - \kappa\epsilon_e\log(\epsilon/\epsilon_e)]\ ,\qquad \epsilon<\epsilon_e\ . \tag{16}$$

The normalization condition then gives[†]

$$1/C_2 = \exp(-\kappa\epsilon_e)\,[1/\kappa + (2\epsilon_e/3)(1 + 2\kappa\epsilon_e/3)]\quad, \tag{17}$$

so that we have, finally,

$$W = [1/\kappa + (2\epsilon_e/3)(1+2\kappa\epsilon_e/3)]^{-1}(\epsilon/\epsilon_e)^{1/2}[1 - \kappa\epsilon_e\log(\epsilon/\epsilon_e)]\ ,$$

$$\epsilon<\epsilon_e;$$

$$W = [1/\kappa + (2\epsilon_e/3)(1 + 2\kappa\epsilon_e/3)]^{-1}\exp[-\kappa(\epsilon - \epsilon_e)],\qquad \epsilon>\epsilon_e\ . \tag{18}$$

We may now calculate the Townsend α for this case; it is

$$\alpha = (1/\mu E)(1/N)\,dN/dt = \sqrt{2/m}\,(N_a/\mu E)\int_{\epsilon_i}^{\infty}\sigma_i W\xi^{1/2}d\xi\quad, \tag{19}$$

where σ_i is the ionization cross section and μ is the mobility. If the distribution decays rapidly enough, we get

$$\alpha \cong \sqrt{2/m}\,(N_a/\mu E)\,\sigma_i(\epsilon_i)\,\epsilon_i^{1/2}\int_{\epsilon_i}^{\infty}Wd\xi\ ,$$

[†] The integral has been extended to $\epsilon = 0$; since $W(\epsilon)$, Eq. (16), vanishes at 0, not much error in normalization should arise from incorrectness of $W(\epsilon)$ in the range $0<\epsilon<\epsilon^*$.

or

$$\alpha \cong (1/K)\sqrt{2/m}\,(N_a/\mu E)\;\sigma_i(\epsilon_i)\;\epsilon_i^{1/2}[1/K + (2\epsilon_e/3)\,(1 + 2K\epsilon_e/3]^{-1}$$
$$\cdot\ \exp[-K(\epsilon_i - \epsilon_e)] \quad .$$

The exponential part is

$$\exp\left[-\left(\frac{N_a\ \sigma_o}{(1/3)e^2\,E^2\lambda}\right)^{1/2}(\epsilon_i - \epsilon_e)\right] \quad ;$$

if we assume that σ_o can be expressed as $\rho\theta$, where $N_a\theta = 1/\lambda$, then this is

$$\exp\,[\,-\sqrt{3}\,\rho\,(\epsilon_i - \epsilon_e)\,/\,eE\lambda\,] \quad ,$$

which differs only slightly from Eq. (3). Note that the quantity $\epsilon_i - \epsilon_e$ appears to the first power. The pre-exponential part is not, however, so simple; it varies either as E or as P^2/E or as a combination of the two, depending on the magnitude of E/P. However, this result is not very significant considering the inexactness of the normalization procedure.

Let us now consider a model in which σ varies linearly with ϵ, i.e. $\sigma = \sigma_o\,(\epsilon - \epsilon_e)$, in contrast to the above. Equations (11) and (14) now describe the course of W, but the normalization must be accomplished by numerical methods. We can obtain certain information about α:

$$\alpha = \sqrt{2/m}\,(N_a/\mu E)\;\sigma_i(\epsilon_i)\;\epsilon_i^{1/2}\;C_3\,i^{4/3}\int_{\epsilon_i}^{\infty}\sqrt{\eta}\,H_{1/3}\,(2i\eta^{3/2}/3)\,d\epsilon \quad .$$

Since one may employ an asymptotic expansion of the Hankel function when its argument is larger than 1,

$$H_{1/3}\,(ir) \cong -\,i^{2/3}\exp(-r)\,(\pi r/2)^{-1/2}\,(1 - 5/72r\ \ldots),$$

one finally obtains for the exponential variation of α

$$\exp\,[\,-(2/3)\,(N_a\,\sigma_o/a)^{1/2}\,(\epsilon_i - \epsilon_e)^{3/2}\,] \quad ,$$

or, if $\sigma_o = \rho\theta$, where $N_a\theta = 1/\lambda$, the exponential is

$$\exp\,[\,-(2\rho^{1/2}/3^{1/2}\,eE\lambda)\quad(\epsilon_i - \epsilon_e)^{3/2}\,] \quad .$$

Note that the dependence on E and P is the same as before (if ρ is constant), but the characteristic energy of the problem is now raised to the 3/2-power. Because the function cannot explicitly be normalized, it is impossible to say anything about the field- or pressure-dependence of the pre-exponential factor. It is clear from the preceding example that no simple dependence over the entire E/P range is to be expected.

It can be shown that, as one writes the excitation cross section to successive powers (σ_o, $\sigma_o\,(\epsilon - \epsilon_e)$, $\sigma_o(\epsilon - \epsilon_e)^2$ . . . .), the exponent of $\epsilon_i - \epsilon_e$ advances by intervals of 1/2, i.e. $(\epsilon_i - \epsilon_e)$, $(\epsilon_i - \epsilon_e)^{3/2}$, $(\epsilon_i - \epsilon_e)^2$ . . . .

III. EXPERIMENTAL EVIDENCE FOR THE TOWNSEND MECHANISM FROM MEASUREMENTS OF α

The function $\alpha(E)$ may be measured directly, as by measurement of prebreakdown current, which depends upon α according to Eq. (1) in the Townsend theory. It may also be determined (up to a multiplicative constant) from breakdown measurements interpreted according to the Townsend theory. Thus, if we assume (Eq. 3) that

$$\alpha/P = A\exp(-BP/E),$$

then the Townsend criterion, $(\omega/\alpha)(\exp\alpha\delta - 1) = 1$, leads to

$$V_s = BP\delta\ /\ (\log P\delta + \log A/K)\ ,$$
$$k\ = \log\ [(1 + \omega/\alpha)/(\omega/\alpha)]\ . \qquad (20)$$

From measurements of sparks, one obtains B and the product A/K. Thus, in any range of E/P in which Eq. (3) applies, one may test the Townsend theory by comparing the goodness of fit to the prebreakdown and to the breakdown data.

Figure 1 shows the equation

$$(\alpha/P) = 7.0\exp(-260\ P/E) \qquad (21)$$

plotted for comparison with the measurements of conduction in N_2 by several observers.(5,14,15) Equation (20), together with these empirical constants and a single determination of (ω/α), should then permit the calculation of the sparking voltage over the whole range of E/P to which Eq. (21) applies. Figure 2 demonstrates the success of such a procedure

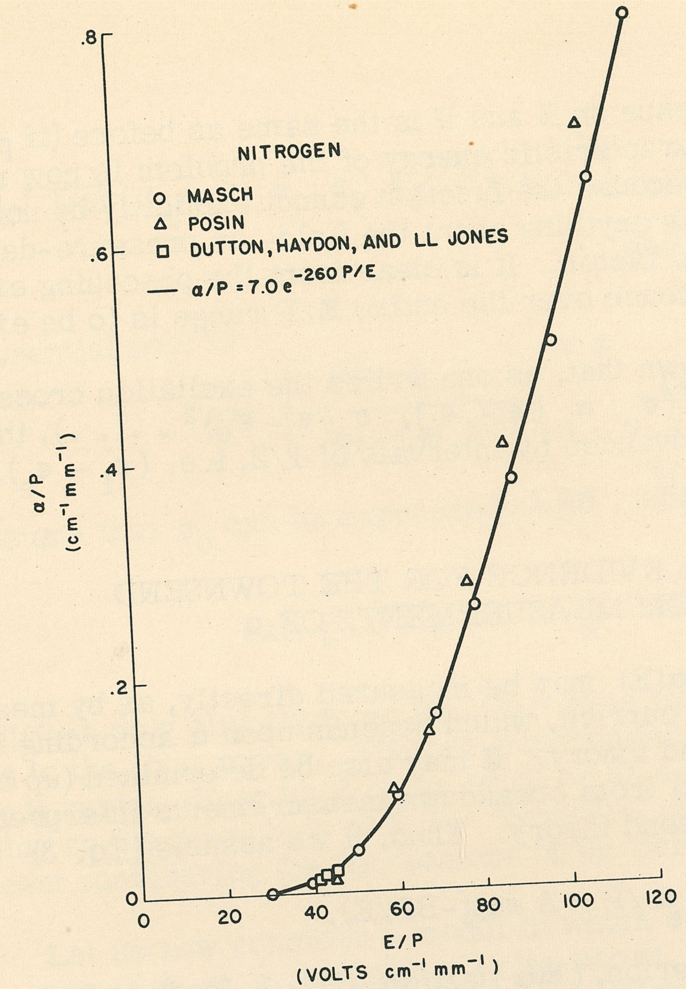

Fig. 1 Comparison of experimental and calculated dependence of α/P upon E/P for nitrogen.

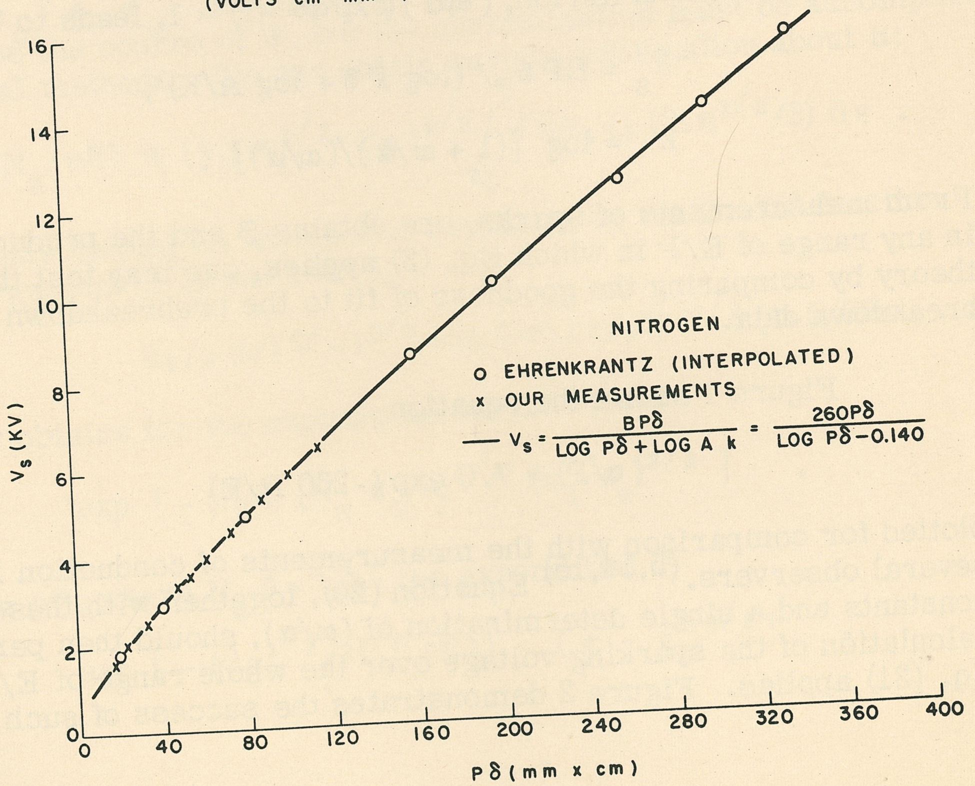

Fig. 2 Comparison of experimental and calculated dependence of V_s upon Pδ for nitrogen.

as applied to the data of Ehrenkrantz(16) as well as to our own, which duplicate those of Ehrenkrantz almost exactly. The value of ω/α so obtained, 3.19×10^{-4}, is in good agreement with recent direct measurements by Dutton, Haydon, and Llewellyn Jones(5) in the region $40 < E/P < 45$.

Figures 3 and 4 show a similar study of conduction data,(17) and application of the results to sparking data,(16) for H_2.

In view of the results of the theoretical study of Section II, it is remarkable that the simple form of Eq. (3) suffices for both N_2 and H_2. It is probably true that the pre-exponential factor is not critically tested by the rather limited range of E/P over which the data are taken.

IV. SPACE-CHARGE FORMATION

The expression for the Townsend current, Eq. (1), is derived assuming that the densities of positive and negative charges are sufficiently low that the electric field is not distorted. Near threshold, this condition clearly no longer obtains, and the formation of space charge must be an essential feature of the mechanism of breakdown, rather than "following after" the spark in some poorly defined way.

We will therefore calculate the Townsend current in the case that space charges are no longer negligible. Varney(18) has done a similar calculation for conduction when $(\omega/\alpha) = 0$ (no secondary mechanism), and our study is essentially an extension of his.

In the steady state, the currents in the gap (cathode at origin, anode at δ) satisfy

$$\begin{aligned} \nabla \cdot i_- &= \alpha i_- , \\ \nabla \cdot i_+ &= -\alpha i_- . \\ i &= i_+ + i_- = \text{constant.} \end{aligned} \tag{22}$$

Furthermore, they are related to the charge densities n_+ and n_- :

$$\begin{aligned} i_- &= n_- e \mu_- E , \\ i_+ &= n_+ e \mu_+ E , \end{aligned} \tag{23}$$

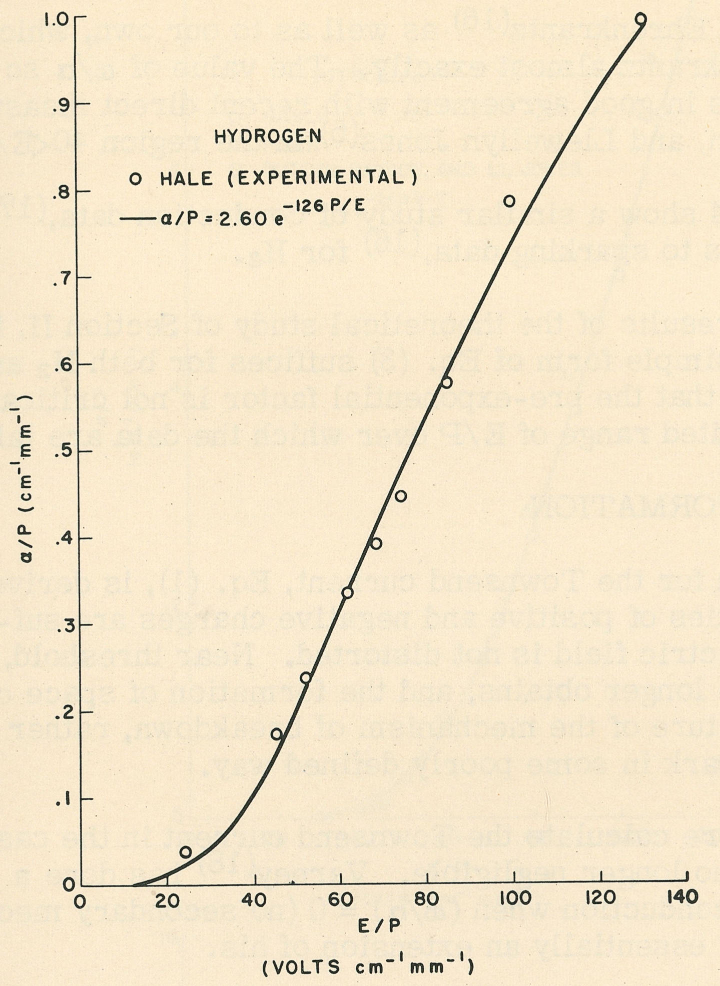

Fig. 3 Comparison of experimental and calculated dependence of α/P upon E/P for hydrogen.

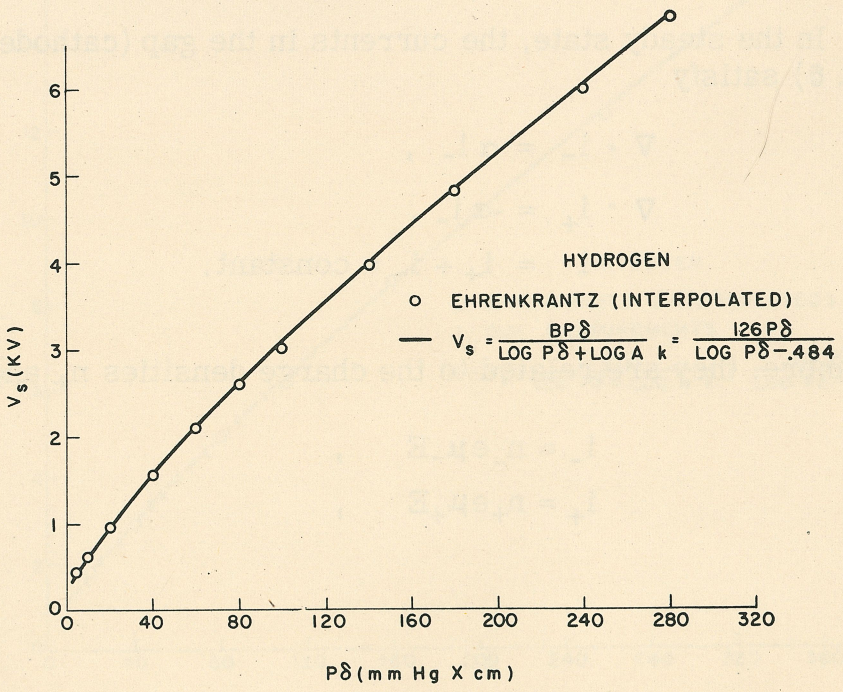

Fig. 4 Comparison of experimental and calculated dependence of V_S upon $P\delta$ for hydrogen.

where μ_- and μ_+ are the magnitudes of the charge mobilities. Finally, n_+ and n_- determine the divergence of the electric field:

$$\nabla \cdot E = 4\pi e\,(n_+ - n_-) \quad . \tag{24}$$

We will specialize these equations to a situation in which the field is directed along the x-axis, and the properties of the system are independent of y and z.

Equations (22) lead to the generalized Townsend equation

$$i_-(x) = \left[1 - (\omega/\alpha)\left(\exp\int_0^\delta \alpha dx - 1\right)\right]^{-1} i_0 \exp\int_0^x \alpha\, dx, \tag{25}$$

if the generalized secondary mechanism characterized by ω/α is introduced. Subsequent operations are facilitated by the introduction of a new independent variable,

$$u = \int_0^x \alpha\, dx,$$

$$du = \alpha\, dx. \tag{26}$$

Then we have

$$i_-(x) = i_0\, e^u / \left[1 - (\omega/\alpha)\,(\exp \bar{u} - 1)\right] \; ,$$

$$\bar{u} = \int_0^\delta \alpha\, dx.$$

Equations (23), (24), and (25) may now be combined to give

$$\alpha\, EdE = 4\pi i_0 \left\{\mu_+ \left[1-(\omega/\alpha)\,(\exp \bar{u} - 1)\right]\right\}^{-1} (\exp \bar{u} - \exp u)\; du. \tag{27}$$

In arriving at Eq. (27), the electron mobility u_- has been considered to be much larger than the ion mobility u_+.

Equation (27) may now be integrated. The quantity α is assumed to be given by Eq. (3). The result is

$$K(1/\xi) = K(1/\xi_0) + (4\pi i_0/s\mu_+)\left[1 - (\omega/\alpha)\,(e^{\bar{u}} - 1)\right]^{-1} \left[u e^{\bar{u}} - e^u + 1\right] \; , \tag{28}$$

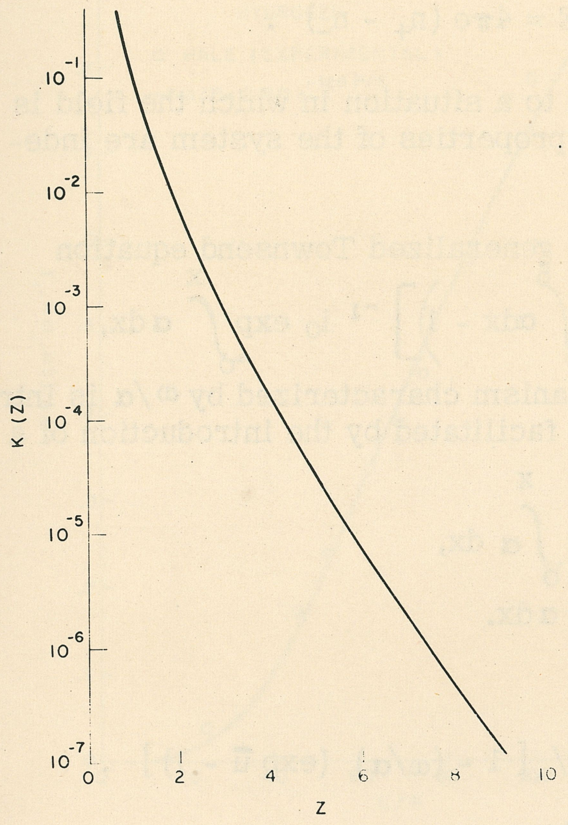

Fig. 5 Calculated curve of K(z) vs z.

where

$$s = AP\,(BP)^2\,,$$
$$\xi = E/BP\,,$$
$$\xi_o = E_o/BP;\quad E_o = E(0),$$
$$K(z) = (1/2)\,[e^{-z}/z^2 - e^{-z}/z - Ei\,(-z)]\,,$$
$$-Ei(-z) = \int_z^{\infty} (e^{-t}/t)\,dt\quad .$$

The function K (z) is plotted in Fig. 5. Then we have for the electric field:

$$E = BP\left\{K^{-1}\,\lfloor K_o + (4\pi\,i_o/s\mu_+)\rfloor\,[1 - (\omega/\alpha)\,(e^{\bar{u}} - 1)]^{-1}\,[u\,e^{\bar{u}} - e^{u} + 1]\right\}^{-1}. \quad (29)$$

Here K^{-1} is the inverse of the K function and $K_o = K\,(1/\xi_o)$. The restrictions on the solution are:

$$\int_0^{\delta} E\,dx = \int_0^{\bar{u}} (E/\alpha)\,du = V, \qquad V = \text{voltage};$$
$$\int_0^{\delta} dx = \int_0^{\bar{u}} (1/\alpha)\,du = \delta, \qquad \delta = \text{gap spacing}. \qquad (30)$$

These restrictions serve to fix K_o and $\bar{u}$; that is, they are two equations relating K_o and $\bar{u}$ that must be solved simultaneously. The two equations are obtained by introducing Eq. (29) into each of the two Eqs. (30). A numerical procedure must be used to obtain the required solution K_o, $\bar{u}$. It involves a trial-and-error determination of pairs (K_o, $\bar{u}$) that satisfy each of the Eqs. (30), so that this intersection may be determined. All the functions of interest, E(x), α(x), $n_+(x)$, $n_-(x)$, etc., are determined by the final solution (K_o, $\bar{u}$).

Equation (29) shows clearly the role of space charges in a Townsend breakdown process. It gives E(x) as essentially equal to E_o when

$$\left|\frac{4\pi i_o}{s\mu_+}\;\cdot\;\frac{u\,e^{\bar{u}} - e^{u} + 1}{1 - (\omega/\alpha)(e^{\bar{u}}-1)}\right| \ll \quad K_o\;.$$

Insertion of physically reasonable numbers shows that in most cases the inequality is violated only when the generalized Townsend threshold defined by

$$(\omega/\alpha)\;(e^{\bar{u}} - 1) = 1$$

is approached. Thus, space charges form when threshold is approached.

For a numerical illustration we have chosen N_2 under the following conditions: P = 700 mm Hg, δ = 1 cm, $(\omega/\alpha) = 3.19 \times 10^{-4}$ (as determined from Eq. 20), B = 260 volts/cm x mm Hg, A = 7.0 cm x mm Hg, $\mu^+ = 3.26$ cm^2/volt x sec, $i_o = 6.36 \times 10^{-11}$ amperes/cm^2. In this case, space charge becomes important only when the voltage is a fraction of one per cent below threshold, showing that the ordinary Townsend condition for threshold is adequate. However, the behavior of current and voltage sheds much light on the mechanism of the spark. Figures 6 through 9 detail the results of calculations for four voltages near threshold. The first three figures show the variation of n_+ (positive ion density), E (electric field), and α (Townsend coefficient) across the gap.

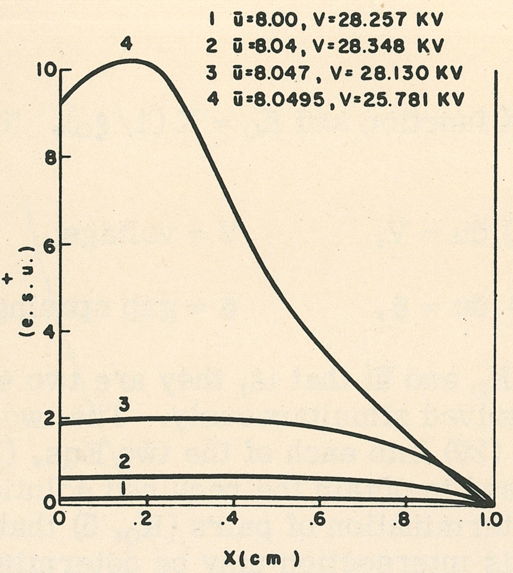

Fig. 6 Distribution of positive ions across the spark gap for various voltages near threshold.

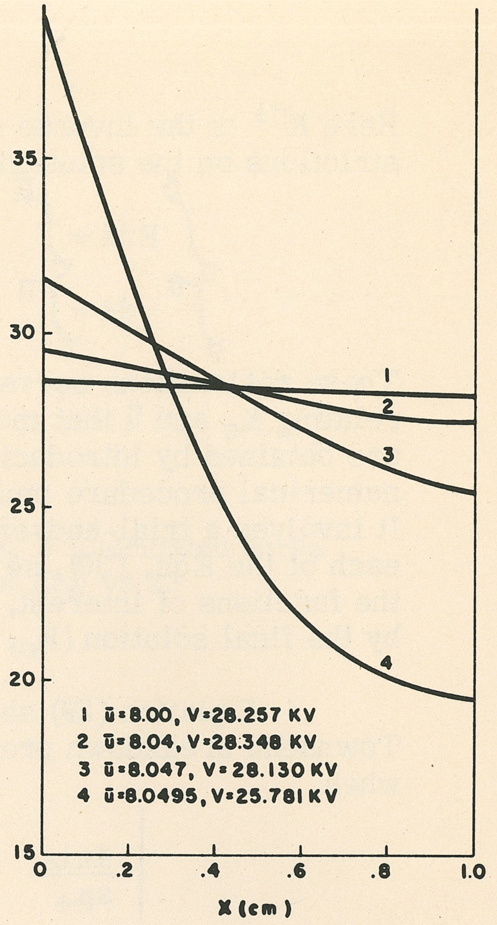

Fig. 7 Distribution of the electric field across the spark gap for various voltages near threshold.

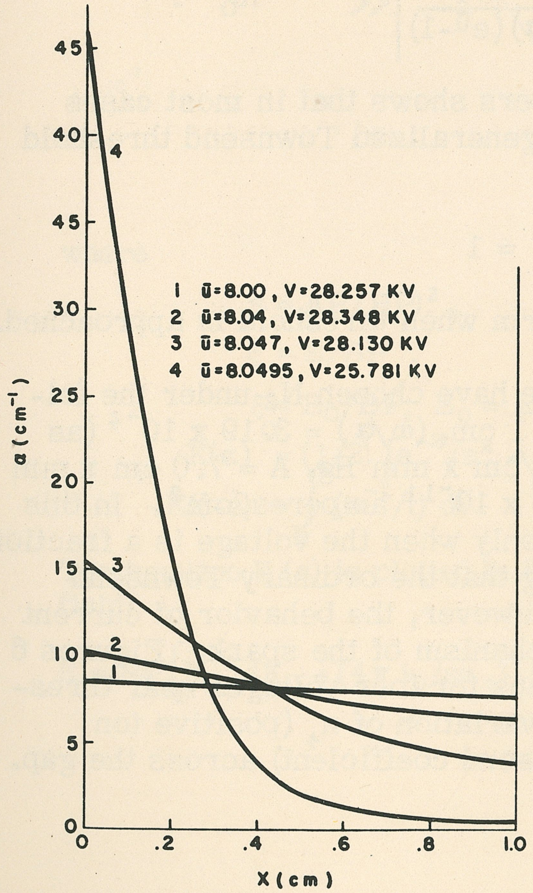

Fig. 8 Distribution of α across the spark gap for various voltages near threshold.

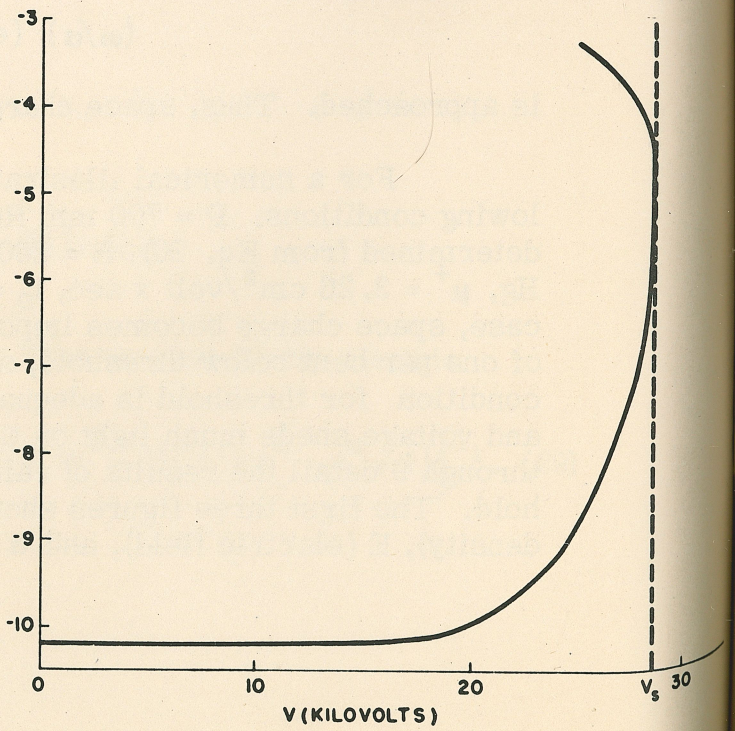

Fig. 9 Variation of the steady state current with voltage.

Note the increasing inhomogeneity. Figure 9 shows the plot of log i vs V thus derived, and includes the approach to threshold described by the ordinary Townsend equation. A fundamental feature of the plot is the negative i-V characteristic which develops beyond i $\approx 3 \times 10^{-6}$ amps/cm^2, illustrating the instability of the system beyond this point.(19) Points on the negative characteristic represent states that are, in principle, steady states; however, they can be measured only if the external circuit is provided with a means of limiting the current to the desired value. That is, the resistance of the gap will drop in an uncontrolled way until redistribution of the major part of the voltage to the external circuit makes possible a steady discharge.

The criterion for breakdown appropriate to this situation is that the slope of the log i vs V curve become infinite, and not that the current itself reach some specified value. That is, if the external circuit is capable of supplying at least the current corresponding to the onset of the negative characteristic, the breakdown is not influenced by the external circuit, but must proceed until the potential drop across the gap is essentially zero.

This runaway property is of course the reason that sparks travel in narrow channels. A suitable condition for a spark in some portion of a spark gap brings about the lowering of resistance in that region and effectively shorts out the other parts of the gap.

It is important to mention that such a negative characteristic can develop only if α increases with some power of E greater than 1; otherwise the function i(V) is single-valued. This has been shown by von Engel and Steenbeck.(19)

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